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Is Minimum Variance Hedging Necessary For Equity Indices?

A Study Of Hedging And Cross-Hedging Exchange Traded Funds

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ABSTRACT

This paper investigates the optimal short-term hedging of Exchange Traded Fund (ETF) portfolios with index futures. Using daily data from May 2000 to December 2004 on the four largest passive ETFs (the Spider, the Diamond, the Cubes and the Russell iShare) and their corresponding index futures we examine the performance of minimum variance hedges for efficient variance reduction and for investors with exponential utility. Our findings relate to daily hedging based on OLS regression, exponentially weighted moving averages and ECM-GARCH models and the utility-based performance evaluation criterion is adopted to capture an efficient reduction in skewness and kurtosis as well as the variance. The basis risk on US equity indices is now extremely low and as a result we find no evidence that minimum variance hedge ratios outperform a naïve 1:1 futures hedge, either for individual ETFs or for portfolios of ETFs. Where minimum variance hedge ratios are useful is for the cross-hedging of ETFs, i.e. the netting of long-short positions prior to placing a futures hedge. We also find that hedging of an ETF portfolio with just one index future can be almost as effective as hedging with all the relevant index futures. Our results should be of interest to tax arbitrage investors in ETFs and their market makers, who often face large and heterogeneous creation and redemption demands on different ETFs. Both types of traders may consider hedging their positions overnight or over a few days.

JEL Classification: C32, G10, G15

Keywords: Exchange Traded Fund, Hedging, Minimum Variance, Utility

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I INTRODUCTION

An exchange traded fund (ETF) is an instrument for investment in a basket of securities. A passive ETF is an index tracking portfolio, like an open-ended index fund but it can be transacted at market price any time during the trading day. We have witnessed a remarkable growth in index ETF trading, particularly in the US during the last decade. At the same time trading has been moving away from the exchange floor towards electronic trading platforms. With the resultant increase in market efficiency, reduced spreads have affected profitable arbitrage opportunities between the ETF, index and futures and these are now very rare and short-lived.¹ Other academic research on index ETFs has examined their price characteristics, the reasons for their underperformance relative to the index and index funds, their tax and cost advantages relative to index funds, the effect of ETFs trading on the liquidity of the underlying stocks and their role in the price discovery process.²

The academic literature on minimum variance hedge ratios has evolved from optimal short-term hedging strategies for commodities, foreign exchange, fixed income and equities, each of them with very different basis risks.³ There is a large literature on minimum variance hedge ratios for hedging equity indices with their index futures, often applying complex models such as the bivariate generalised autoregressive conditional heteroscedasticity (GARCH) model with maturity effects captured through the disequilibrium term in an error correction model of spot and futures returns.⁴ But the basis risk between the ETF and future is now so low that minimum variance hedge ratios may not perform significantly better than a naïve 1:1 futures hedge.⁵

The aim of this paper is to examine the hedging decision facing market makers and other short-term traders in ETFs, including specialists acting as principals, who may take large overnight positions on their own account. We investigate the effectiveness of minimum variance hedging with futures, the extent to which a long position on one index ETF is hedged by a short position on another correlated index ETF and we determine the optimal mix of futures for hedging ETF portfolios. The remainder of the paper is structured as follows: Section II describes the ETF market characteristics; Section III analyses the empirical properties of mispricing and basis risk; Section IV provides a comparison of different minimum variance hedge ratios for hedging individual index ETFs with index futures; Section V examines the extent to which the risk of one ETF can be hedged by an opposite position in a correlated ETF; Section VI investigates how best to hedge a portfolio of index ETFs using the most liquid index futures and Section VII summarizes and concludes.

¹ See Switzer, Varson, and Zghidi (2000), Akhert and Tian (2001), Chu and Hsieh (2002), and Kurov and Lasser (2002).

² See and Chu, Hsieh and Tse (1999), Akhert and Tian (2000), Elton *et al.* (2002), Poterba and Shoven (2002), Kostovetsky (2003), McDermott and Hegde (2006) and Gastineau (2004).

³ See Cecchetti, Cumby and Figlewski (1988), Baillie and Mayers (1991), Kroner and Sutan (1991) and Lin, Najand and Yung (1994).

⁴ See Hill and Schneeweis (1984), Figlewski (1984, 1985), Junkus and Lee (1985), Peters (1986), Graham and Jennings (1987), Merrick (1988), Lindahl (1991, 1992), Bera, Bubnys and Park (1993), Stoll and Whaley (1993), Benet & Luft (1995), Park and Switzer (1995), Geppert (1995), Lien, Tse & Tsui (2002), Brooks, Henry and Persaud (2002), Miffre (2004) and many others. A useful survey of this work is given in Sutcliffe (2005).

⁵ See for instance Alexander and Barbosa (2005).

II THE MARKET AND CHARACTERISTICS OF ETFs

The main characteristics of ETFs are their low cost structure, the in-kind creation and redemption of shares, arbitrage pricing mechanisms, tax advantages and secondary trading of shares. Two main features allow index ETFs to present a low cost structure: the passive management role of the trustee and the absence of shareholder accounting at the fund level. Since brokerage firms and banks manage shareholder accounting the ETF trust does not need to keep records of the beneficial owner of its shares and this represents an important cut in the fund's cost structure. On the other hand, ETF trading may have brokerage and commission fees that an investor does not face when acquiring or redeeming mutual fund shares.

The in-kind redemption and creation of shares is the core characteristic that allows ETFs to be cost efficient, by avoiding excessive turnover of portfolio securities otherwise needed to attend creations and redemptions. Shares of the ETFs can be created and redeemed in block-size 'creation units' on a daily basis, with the deposit of the portfolio securities and a cash component corresponding to dividends and other expenses.⁶ The fund delivers to the redeeming shareholder low cost securities in-kind and thus taxable capital gains are also relatively low. But aside from the tax advantages the in-kind redemption and creation of ETF shares allow arbitrage between the stocks and the fund's shares, ensuring that the market price of the fund does not deviate too far from its net asset value (NAV). If the fund's price rises too far above its NAV the market maker, acting as arbitrageur, may buy stocks to create new units of the fund; and if the fund's price falls too far below the NAV the market maker may redeem units of the fund for the constituent stocks.

The first successful ETF, the Standard and Poor's Depository Receipt (SPDR – pronounced 'Spider') was released by the American Stock Exchange (AMEX) in 1993. The SPDR Trust is a unit investment trust designed to correspond to the price and yield performance of the S&P 500 Index. The objective of this innovative exchange traded unit trust was to allow intra-day trades on an indexed portfolio basket. The Spider is now one of the most widely traded ETFs with about 55 billion US\$ under management as of December 2004, representing over 24% of US market in passive ETFs. By the end of 2004 there were 151 ETFs in the American market with assets under management of over 226 billion US\$.⁷

ETFs offer investors many benefits of exchange trading such as short selling, limit orders and exemption from the up-tick rule that prevents short selling except after a price increase. Other benefits include relatively low trading costs and management fees, diversification, tax efficiency and liquidity. Consequently since the inception of the Spider in 1993 the average annual growth in assets under management by passive ETFs was an impressive 85%, and the growth rates of both the number of funds and their assets

⁶ In the case of the Russell 2000 iShare the portfolio of securities are closely approximating the holdings of the Fund. In other ETFs that we study the investor may deposit 115% of the market value of undelivered securities.

⁷ Source: 2005 Investment Company Fact Book.

under management have outperformed the corresponding growth rates in the mutual fund industry. Yet by December 2004 ETFs only accounted for 2.79% of the total mutual funds industry. Clearly investment in ETFs is set to rise very significantly in future.

We shall examine the risks of trading and market making in four funds that by the end of 2004 together accounted for 40% of the assets invested in US passive ETFs. These are:

- The ‘Spider’, i.e. the S&P500 SPDR that was listed on the AMEX in 1993: ticker symbol SPY. It remains by far the largest passive ETF with 54.83bn\$ under management by December 2004. The Spider share price corresponds to 1/10th of the S&P500 index value.
- The ‘Cubes’, i.e. the Nasdaq-100 ETF: ticker symbol QQQQ. This is the second largest ETF in the US, launched in March 1999 and by December 2004 having 20.36bn\$ under management. The Cubes share price is approximately 1/40th of the Nasdaq-100 index value.
- The ‘Diamond’, i.e. the ETF tracking the Dow Jones Industrial Average (DJIA) index: ticker symbol DIA. It began trading in January 1998 and by December 2004 had 7.74bn US\$ under management. The Diamond share price is approximately 1/100th of the DJIA index value.
- The Russell 2000 iShare: ticker symbol IWM. This was launched in May 2000 and had 6.55bn US\$ under management by December 2004. The Russell iShare price corresponds to 1/5th of the Russell 2000 index value.

All four trusts issue and redeem shares in creation units of 50,000.

Figure 1 shows how the total market values of these funds evolved after May 2000, when the Russell iShare began trading, until December 2004. During this period US equity markets have been characterized by high volatility and low returns, hence the growth in the Spider’s market value represents a significant increase in shares outstanding. The other funds’ market values have not grown as much and the Cubes in particular had not increased in market value at all since January 2001. The number of Cubes shares outstanding has increased, but the Nasdaq-100 index fell by 37.5% between May 2000 and January 2001 and by 26.5% between January 2001 and December 2004.

[Figure 1]

The treatment of dividends has a direct influence on the creation and redemption of shares for tax management purposes (Gastineau, 2002). The holder of the ETF on the ex-dividend date is entitled to receive the dividends, no matter how long the share has been held. But if the share is sold during the ex-dividend period the registered investor loses the dividends and any tax advantage or disadvantage related to it. Moreover ETFs traded on the secondary market do not include the dividend or cash components. Hence there is considerable scope for tax arbitrage around the time of dividend payments.

Figure 2 graphs the net daily creation and redemption series for each fund as a percentage of its NAV. Note that very large daily net creations or redemptions of around 5% of the NAV of the fund are quite

normal and it is not uncommon for redemption or creation demand to be over 10% of NAV. For the tax reasons mentioned above creations and redemptions are particularly active around the dividend dates, especially for the Spider and Diamond as these pay significant dividends.

[Figure 2]

Table 1 compares the daily average of net creations and redemptions of the total sample with the daily average around the ex-dividend dates (which varies from the 13th to the 20th of each of the dividend months, i.e. March, June, September and December). The positive mean in each case is a result of the huge net creation of ETF shares over the period. The standard deviation measures the extent of creation/redemption activity. The middle section of Table 1 examines the creation/redemption activity around the dividend dates. The regular quarterly ex-dividend date for the Spider and the Cubes is the third Friday in each of March, June, September and December. However, from inception until the end of 2004 the Cubes paid dividends only twice, in December 2003 and December 2004. The Diamond has monthly dividend payments and the dividend stream of the Russell iShare, although quarterly, does not coincide with that of the Spider.

TABLE 1: NET DAILY CREATIONS AND REDEMPTIONS⁸

Net Daily Creations and Redemptions (Total sample)				
	SPY	DIA	QQQQ	IWM
Mean	0.123%	0.135%	0.129%	0.320%
StDev	1.273%	2.167%	1.321%	2.809%
Net Daily Creations and Redemptions (Around dividend dates only)				
	SPY	DIA	QQQQ	IWM
Mean	0.629%	0.649%	0.012%	0.318%
StDev	2.163%	2.352%	2.592%	1.545%
Correlations of Net Daily Creations and Redemptions				
	SPY	DIA	QQQQ	IWM
SPY	1	-0.07211	0.03996	-0.00908
DIA		1	0.01006	0.02680
QQQQ			1	0.03352
IWM				1

We find a marked increase in creation/redemption activity around dividend dates for the three largest funds, but less activity in the Russell iShare. Both the Spider and the Diamond have large creation demands before dividends are paid and there is evidence that investors move their normal creation demand away from the Cubes and into these funds to receive the higher dividend of the Spider and the Diamond. The lower part of Table 1 displays very low correlations between the creation/redemption series of the four funds and this indicates that market makers are likely to face demand for either long or short positions in different ETFs. The funds clearly have quite different distribution streams and the demand and supply of ETFs shares is quite heterogeneous.

⁸ Expressed as a percentage of the number of shares outstanding on the previous day.

III MISPRICING AND BASIS RISK OF ETFs

The previous section shows that several factors may contribute to a price difference between the ETF and the spot index and that market makers perform a central role in reducing the ‘mispricing’ in ETF markets by ETF-index arbitrage. Another possibility is to arbitrage the fund with the index future. The effect of using an ETF in place of an index for futures arbitrage is to reduce the no-arbitrage range for the future, compared with that based on the index. When the future is sold and the spot index is bought, and even more so when hedge portfolio is long the future and short the index, the trading costs from dealing individual securities are high. These present a barrier to arbitrage, and the no-arbitrage range for the market price of the future about its fair price is relatively wide. However, costs are significantly reduced when the ETF is used in place of a portfolio replicating the index. Moreover like futures, ETFs are not held to the up-tick rule so short arbitrage is also easier with an ETF. Consequently the no-arbitrage range for the market price around the fair price of the future is smaller in the presence of an ETF as an arbitrage vehicle and in particular the incidence of negative mispricing, where the market price of the future is much less than the fair price, is reduced.

To demonstrate this, write the market price of the T -maturity index future at time $t < T$ as

$$F_t = F_t^* + x_t S_t \quad (1)$$

where

$$F_t^* = \exp((r - q)(T - t)) S_t \quad (2)$$

is the theoretical or ‘fair’ value of the future based on the ETF price S_t and the risk-free T -maturity interest rate r and dividend yield q on the fund are both assumed to be non-stochastic. Many authors refer to x_t as the ‘mispricing’ of the market price of the future compared with its fair value but it is really the spot rather than the future that is mispriced because it is the future that serves the dominant price discovery role. The average mispricing of the fund relative to the future depends on the handling of dividends and the transactions costs, as we shall see below.

The variance and higher moments of the mispricing series represents the basis risk that might be hedged using a hedge ratio different from the ‘naïve’ 1:1 futures hedge. To see this, consider a cash position at time $t = 0$ with value S_0 that is hedged by selling β units of a T -maturity future with market price F_0 and suppose the position is closed at time τ , with $0 < \tau < T$. The change in value of the hedged portfolio is

$$S_\tau - S_0 - \beta(F_\tau - F_0)$$

so that, at any time t , with $0 < t < \tau$, the value of the hedge position is:

$$v_t = S_t - \beta(F_t - F_0) = S_t + \beta F_0 - \beta S_t (x_t + (1 - b_t^*)) \quad (3)$$

where

$$b_t^* = \frac{S_t - F_t^*}{S_t} = 1 - \exp((r - q)(T - t)) \quad (4)$$

is the fair value of the basis as a proportion of the cash price.

In the expression (3) we have chosen to single out the fair value of the basis b_i^* as a separate term. This is because the discount rate and the dividend yield have much less uncertainty than other determinants of the value of the hedged portfolio. The formulation (3) allows one to extract the effect of the variability in the fair basis (4), which is largely deterministic, from the real uncertainty that needs to be hedged. If discount rates and dividend yields are deterministic the basis risk is only due variations in the mispricing x_i . Hence in the following we ignore the large and predictable movements in the fair value of the basis and use the volatility, skewness and kurtosis of the mispricing as indicators of basis risk.

Bloomberg closing daily prices from May 2000 to December 2004 were obtained on the Spider, Cubes Diamond and Russell iShare. All portfolios studied in this paper were based on a block size of ETFs corresponding to one unit of the underlying index in order to match the futures contract trading unit based on the spot value of the index. That is, for each trade unit we hedge 10, 100, 40 and 5 shares of Spider, Diamond, Cubes and iShares, respectively. Note that trading in both the fund and the future ceases at 4:15 p.m. EST. Hence the mispricing series is a true representation of the deviation about fair value. By contrast the index closes 15 minutes earlier than the future, so hedging studies on daily index close prices necessarily analyse non-synchronous data and this may have introduced some bias in the results of certain previous studies.

Following Ackert and Tian (2000) we have adjusted each fund's price by deducting the value of the cash component. The Spider, Cubes and Russell iShare pay quarterly dividends that coincide with the date of the expiration of the futures. Hence there is no dividend uncertainty included in the arbitrage relation between the fund and the index future as all dividends, expected and paid, are isolated in the cash account. This is not true for the Diamond as it pays monthly dividends. For this reason, besides the cash component adjustment made to all four funds, we also adjusted the Diamond theoretical futures price for dividends paid before the expiration of the futures contract.

Table 2 shows that our hedging results will cover two quite distinct two-year periods in the US equity market: the bear market from January 2001 until December 2002 and the recovery phase from January 2003 until December 2004. All four funds performed badly over the first period and this period is by far the most volatile, as it covers the aftermath of the technology bubble and the terrorist attack on the US. The period 2003-2004 was much less volatile, as markets began to recover the losses made between 2000 and 2002.

TABLE 2: DESCRIPTIVE STATISTICS OF THE ETF RETURNS

2001-2002	SPY	QQQQ	DIA	IWM
Average annual return	-20.68%	-44.84%	-12.87%	-12.07%
Volatility (annualized)	24.67%	48.64%	24.27%	25.79%
Skewness	0.19	0.35	0.05	0.06
XS Kurtosis	0.81	1.46	2.20	0.09
2003-2004	SPY	QQQQ	DIA	IWM
Average annual return	16.09%	24.58%	12.96%	27.20%
Volatility (annualized)	14.21%	21.40%	13.71%	18.69%
Skewness	-0.04	-0.06	0.05	-0.22
XS Kurtosis	0.98	0.74	1.39	-0.40

In both periods the Diamond, being based on Blue Chip stocks, was the least volatile and the Cubes the most volatile, reflecting continued uncertainty surrounding performance of technology stocks following the burst of the technology bubble. A higher volatility in the Russell 2000 iShare is also to be expected, as it has the lowest market capitalization and is also the most recently issued of the four funds. Apart from this the higher moments indicate the heavy-tailed and slightly skewed nature of the fund's returns distributions.⁹

TABLE 3: DESCRIPTIVE STATISTICS OF MISPRICING

2001-2002	SPY	DIA	QQQQ	IWM
Mean (daily)	-0.52%	-0.10%	0.42%	0.26%
Volatility (annualized)	2.24%	2.78%	3.04%	4.64%
Skewness	0.1819	2.3742	-2.5267	0.4498
XS Kurtosis	1.3830	16.3531	30.4786	2.1058
2003-2004	SPY	DIA	QQQQ	IWM
Mean (daily)	-0.62%	-0.13%	0.50%	0.10%
Volatility (annualized)	1.84%	1.67%	1.66%	3.20%
Skewness	1.9259	0.2682	0.8846	0.8376
XS Kurtosis	7.9143	6.1919	7.6067	2.0525

Table 3 reports the sample statistics each funds' mispricing relative to the future over the two sub-samples. As explained above, the volatility and higher moments of the mispricing series captures the extent of the basis risk. Already small in the first period, the volatility was even lower in the 2003-4 period.

⁹ The standard error is approximately $\sqrt{6/T}$ for the skewness and $\sqrt{24/T}$ for the excess kurtosis where T is the sample size. In our case, with T approximately equal to 500 in each sub-sample, the approximate standard error for the skewness coefficient is 0.11 and for the excess kurtosis it is approximately 0.22. Note that the excess kurtosis is significantly different from zero (except for the IWM) but that it was at a relatively low level compared with the 1990s. For instance, from 1993 until December 2004 the sample excess kurtosis of the S&P 500 index daily returns was 3.69, having achieved a maximum of 10.91 during September 1998, although over the entire period 2001-2004 the excess kurtosis of the index was only 1.88.

At less than 2% p.a. for the three more established funds and only 3.2% for the Russell iShare we may expect that minimum variance hedge ratios will be very close to the naïve 1:1 futures hedge. But note that the large (but usually positive) skewness and very significant excess kurtosis of the mispricing series indicates that any hedge could fail spectacularly on some days. We shall consider both these questions in more detail in the next section.

Figure 3 plots the mispricing series of the future relative to the ETF. At the beginning of the period mispricing was relatively large and volatile on all funds, especially on the Cubes due to the excessive volatility in Nasdaq-100 shares and the Russell iShare, which had only just been launched. Overall the largest positive mispricing has been on the Cubes and the largest negative mispricing has been on the Spider. Since January 2002 these mispricing series have remained very stable, being around +50bps for the cube and around -60bps for the Spider. Why does this small but persistent mispricing arise in these funds?

[Figure 3 here]

Table 4 shows that the two funds with negative mispricing (the Spider and Diamond) have the highest dividend yield. That is, even after our cash account adjustment these funds are being priced at a premium. But the same two funds also have the lowest turnover and the lowest expense ratios. Clearly the sign of the mispricing can be related to trading costs: the Cubes and iShare, which are normally priced at a discount to their index, have higher trading costs; the Spider and Diamond, which are normally priced at a premium to their index, have lower costs.

TABLE 4: DIVIDEND YIELD, EXPENSE RATIO AND TURNOVER

	SPY	DIA	QQQQ	IWM
Benchmark Index Dividend Yield (On average 2000/2004)	1.54%	1.98%	0.32%	1.30%
ETF Expense ratio (2004)	0.10%	0.18%	0.20%	0.20%
ETF Portfolio Turnover (2004)	2.23%	3.88%	6.60%	20.00%

IV HEDGING ETFs WITH INDEX FUTURES

The short-term hedging of index ETFs is of particular interest to tax arbitrage investors, as we have seen that ETF trading is particularly active around the time of dividend payments. It is also of interest to ETF specialists, i.e. members of the exchange that are the designated market makers in the ETF. The main ETF market makers in the US are Spear, Leeds and Kellogg, Susquehanna International Group, the Hull Trading Company and Bear Hunter. They are responsible for maintaining a liquid and continuous two-sided auction market and ensuring that markets are fair, orderly and competitive by acting both as an

agent and a principal. They publicly quote and transact firm bid and offer prices, making money on the spread, and buy or sell on their own account to counteract temporary imbalances in supply and demand. This stabilizes prices but then, as principal dealers, the specialists bear the market risk.

Despite the increased competition from electronic trading, specialists remain key players in the market for ETFs for a number of reasons. Specialists facilitate competitive pricing for trading new or illiquid ETF products such as active ETFs (which seek to out-perform an index by deviating from the passive portfolio) or ETF futures and options. Finally there is some evidence that the performance of electronic trading systems deteriorates during periods of intense activity, in that bid-offer spreads are more sensitive to price volatility in electronically traded markets (see Aitken *et al.* 2004). Hence specialists are also necessary to facilitate smooth trading during volatile periods of liquidity shortage. But with electronic platforms moving trading away from the exchange floor spreads are considerably reduced, profits are squeezed and market specialists clearly need to focus on hedging their risks in an optimal manner.¹⁰

Our investigation of minimum variance short-term hedge ratios for index ETFs uses the closing price of the future as the transaction price for the hedge. That is because market orders for creation and redemption may be placed until 4:00 p.m. New York time and at this time the market maker needs to decide whether to create or redeem shares, to keep an open position on their own account, or to hedge their open positions in other markets. If they choose to hedge with the future then the hedge would be effected at 4:15 p.m., or just before.

We construct several portfolios comprised of a spot position in the ETF and a short position in the index futures. We report results for hedging the spot ETF with the index futures using the 1:1 hedge ratio and time-varying minimum variance hedge ratios obtained using three different econometric models: ordinary least squares (OLS) with a rolling in-sample estimation periods of six months,¹¹ exponentially weighted moving average (EWMA) with a smoothing constant of 0.95, and error correction regression with multivariate generalised autoregressive conditionally heteroscedastic errors (ECM-GARCH).¹² The appendix gives details of the method used to calculate each hedge ratio.

¹⁰ This has motivated the development of new futures contracts on ETFs launched on the Chicago Mercantile Exchange on June 6, 2005. These contracts are based on three of the four funds examined in this paper: the Spider, the Cubes and the Russell 2000 iShare. Futures on Diamond are traded at OneChicago, an electronic exchange based on a joint venture between: the Chicago Board Options Exchange (CBOE), Chicago Mercantile Exchange Inc. (CME) and the Chicago Board of Trade (CBOT).

¹¹ We also used one-year estimation period for the OLS hedge ratio and the results were very similar to the 6-months OLS hedge ratios, the latter performing slightly better on specific occasions. The difference however is not statistically significant. We report only the 6-month results so as to cover the longest period in our results. Results for the 1-year OLS hedged portfolio are available from the authors on request.

¹² A variety of bivariate GARCH(1,1) parameterisations of the dynamics of \mathbf{H}_t were explored including several BEKK specifications (Engle and Kroner, 1995). The BEKK specification ensures positive definiteness while imposing cross equation restrictions (e.g. the scalar BEKK imposes that persistence in volatility and correlation are the same). We also used the t -BEKK, which replaces the conditional normality assumption with that of conditionally t -distributed error terms, and the dynamic conditional correlation (DCC) model of Engle (2002). The results for these methods were very similar, with no implications for the final conclusions so we only report the diagonal BEKK results in this paper. However, the results for the other GARCH models are available from the authors by request.

Each day we estimated the hedge ratio based on a rolling in-sample period, which determines the futures position to be taken at the end of the day until the following day. The sample is then rolled one day, the hedge ratios re-estimated, and the hedge re-balanced and held until the end of the next day. We thus form an ‘out-of-sample’ hedge portfolio returns series. Since the minimum variance criterion is applied in-sample and the hedging performance is tested out-of-sample there is no guarantee that minimum variance hedging will produce more effective hedges than the unconditional 1:1 futures hedge.

Hedging performance will be measured in two ways. First we use the proportional variance reduction measure proposed by Ederington (1979): denoting by V_U and V_H the variance of the un-hedged portfolio returns and the variance of the hedge portfolio out-of-sample returns respectively, this measure of hedge performance, which is termed the ‘effectiveness’, E in our results, is given by:

$$E = \frac{(V_U - V_H)}{V_U} \quad (5)$$

The Ederington effectiveness E is widely used even though it is known to favour the OLS hedge (see Lein, 2005). Also it takes no account of the effect of variance reduction on skewness and kurtosis. Minimum variance hedged portfolios are designed to have very low returns volatility and this could increase an investor’s confidence to the extent that large leveraged positions are adopted. However the higher moments of hedged portfolio returns can indicate cause for concern: a high kurtosis indicates that the hedge can be spectacularly wrong on just a few days and a negative skewness indicates that it would be losing rather than making money.

Following Scott and Horvath (1980), Cremers et al. (2004), Harvey *et al.* (2004), Patton (2004) and others our second measure of hedge effectiveness accounts for skewness and kurtosis in out-of-sample performance of hedged portfolios. It is natural to base utility on an investor’s level of wealth although it may be more intuitive empirically to use the moments of portfolio returns in the certainty equivalent (CE), as for instance in Harvey *et al.* (2004). We thus compute the CE derived from an exponential utility for the hedger, based on both the portfolio’s out-of-sample returns and by constructing an out-of-sample time series of profits and losses (P&L) from an investment of 1 million US\$ in all portfolios considered. The exponential utility function is:

$$U(x) = -\lambda \exp(-x/\lambda) \quad (6)$$

where x is wealth and λ is the coefficient of risk tolerance, which defines the curvature of the utility function and which is measured in the same units as wealth. The CE is that level of wealth such that $U(x) = E[U(x)]$ where $E[U(x)]$ is the expected utility associated with a profit and loss distribution.

Applying the expectation operator to a Taylor expansion of $U(x)$ about $U(\mu)$, where $U(\mu)$ is the utility associated with the mean P&L (or mean return) provides a simple approximation for the CE associated with any utility function:

$$E[U(x)] = U(\mu) + U'(x)|_{x=\mu} E[x - \mu] + \frac{1}{2} U''(x)|_{x=\mu} E[(x - \mu)^2] + \frac{1}{3!} U'''(x)|_{x=\mu} E[(x - \mu)^3] + \dots$$

With $U(x)$ defined in (6) and setting $x = CE$ the above gives an approximation:

$$\exp(-CE/\lambda) \approx \exp(-\mu/\lambda) \left(1 + \frac{E[(x - \mu)^2]}{2\lambda^2} - \frac{E[(x - \mu)^3]}{6\lambda^3} + \frac{E[(x - \mu)^4]}{24\lambda^4} \right)$$

Thus the certainty equivalent associated with the exponential utility function is approximated as:

$$CE \approx \mu - \frac{\sigma^2}{2\lambda} + \frac{\phi}{6\lambda^2} - \frac{\kappa}{24\lambda^3} \quad (7)$$

where μ and σ are the mean and the standard deviation of x , $\phi = E[(x - \mu)^3]$ and $\kappa = E[(x - \mu)^4]$.

The formulation (7) shows that when the risk tolerance parameter $\lambda > 0$ there is an aversion to risk associated with increasing variance, negative skewness and increasing kurtosis. In order to capture higher moment effects we have chosen to calculate CE based on the sample moments of the relevant out-of-sample daily returns using $\lambda = 10\%$ and the out-of-sample daily P&L using $\lambda = 500$.¹³

TABLE 5: PERFORMANCE OF FUTURES HEDGES: 2001-2002

Portfolio		Moments of Returns				Performance Measures		
		Mean	Volatility	Skewness	XS Kurtosis	E	CE Returns	CE P&L
SPY	Naïve 1:1 Hedge	1.12%	2.53%	-1.1096	9.8709	98.95%	75.48	69.22
	OLS	1.31%	2.54%	-1.0689	10.1938	98.94%	94.45	68.9
	EWMA	1.21%	2.57%	-0.9078	9.6545	98.91%	84.03	68.27
	ECM-BEKK	1.34%	2.58%	-1.0618	9.4022	98.91%	96.46	67.97
DIA	Naïve 1:1 Hedge	1.11%	2.11%	-1.243	19.67	99.24%	85.51	72.72
	OLS	1.14%	2.14%	-1.3235	20.4537	99.22%	86.67	69.32
	EWMA	1.44%	2.17%	-1.2918	19.8465	99.20%	116.54	70.21
	ECM-BEKK	1.17%	2.13%	-1.2827	19.3142	99.23%	90.47	71.99
QQQQ	Naïve 1:1 Hedge	0.46%	3.63%	-1.0451	13.8077	99.44%	-38.02	-68.84
	OLS	0.60%	3.65%	-1.0318	13.6031	99.44%	-24.95	-73.59
	EWMA	0.38%	3.68%	-0.9419	13.2188	99.43%	-47.89	-72.71
	ECM-BEKK	0.41%	3.64%	-1.005	13.733	99.44%	-43.08	-70.9
IWM	Naïve 1:1 Hedge	0.60%	5.80%	-0.1203	3.5268	94.94%	-128.58	-304.75
	OLS	0.27%	5.82%	-0.2554	3.1432	94.91%	-165.33	-268.02
	EWMA	-0.75%	5.88%	-0.2872	2.9065	94.81%	-271.5	-297.78
	ECM-BEKK	-0.05%	5.86%	-0.1843	3.189	94.84%	-198.21	-298.42

¹³ Since utilities are only unique up to positive affine transformations it is admissible to apply a linear transformation to the result provided the transformation is the same for all series that are being compared, and we have done this merely to present the CE figures on an intuitive scale.

TABLE 6: PERFORMANCE OF FUTURES HEDGE: 2003-2004

Portfolio	Moments of Returns				Performance Measures		
	Mean	Volatility	Skewness	XS Kurtosis	E	CE Returns	CE Percentile
Naïve 1:1 Hedge	-0.42%	1.87%	-1.9338	19.3213	98.27%	-62.46	82.47
SPY							
OLS	-0.45%	1.88%	-1.8583	19.284	98.25%	-65.46	81.79
EWMA	-0.44%	1.91%	-1.517	19.3277	98.20%	-65.26	82.20
ECM-BEKK	-0.63%	1.90%	-1.7087	18.3514	98.21%	-83.65	82.45
Naïve 1:1 Hedge	-0.25%	1.15%	-0.0818	7.4595	99.30%	-31.86	98.50
DIA							
OLS	-0.22%	1.15%	-0.0883	7.5662	99.30%	-28.85	98.47
EWMA	-0.33%	1.16%	-0.1068	7.2365	99.28%	-39.75	98.48
ECM-BEKK	-0.25%	1.16%	-0.0979	7.201	99.28%	-31.81	98.48
Naïve 1:1 Hedge	-0.60%	2.33%	-0.7397	5.2917	98.81%	-89.28	83.37
QQQQ							
OLS	-0.47%	2.33%	-0.7647	5.237	98.81%	-76.91	82.93
EWMA	-0.47%	2.35%	-0.7786	5.1331	98.80%	-76.50	83.42
ECM-BEKK	-0.76%	2.35%	-0.7526	5.1559	98.80%	-106.05	83.46
Naïve 1:1 Hedge	-0.04%	3.22%	-0.1893	4.4325	97.04%	-58.47	53.94
IWM							
OLS	0.07%	3.22%	-0.157	4.4659	97.03%	-48.14	53.36
EWMA	-0.12%	3.24%	-0.1879	4.4104	96.99%	-67.81	53.41
ECM-BEKK	-0.12%	3.23%	-0.1823	4.5465	97.02%	-66.92	53.03

According to the Ederington effectiveness criterion our results in Tables 5 and 6 show that the most effective hedges (with over 99% variance effectiveness in both periods) are obtained when hedging the Cubes and the Diamond and the least effective hedge (less than 95% variance effectiveness in the 2001-2002 period) is on the Russell iShare. The hedges also effectively neutralize the large negative mean returns to the ETFs during 2001-2002 and the large positive mean returns during 2003-2004. Note that the minimum variance hedge ratios never achieve more effective variance reduction than the 1:1 futures hedge: this is seen for all ETFs and in both sub-samples.¹⁴ There is weak evidence that minimum variance hedging produces portfolios with less negative skewness and lower kurtosis because the CE is not always maximized using the 1:1 hedge. However in the few instances when the preferred portfolio is a minimum variance hedged portfolio, which of the OLS, EWMA or ECM-BEKK hedges is best depends on the time period and the ETF considered. Overall, the utility-based results for hedging performance corroborate those of the Ederington effective criterion and there is no clear evidence that minimum variance hedge ratios can improve on the 'naïve' 1:1 futures hedge strategy.

¹⁴ In these and the following Tables bold type is used to highlight the best performance where applicable.

V CROSS-HEDGING ETFs

When matching creation and redemption baskets the market maker is likely to have a large net creation or redemption demand on each fund at the end of a day. It is not uncommon for daily net creation-redemption demands to be over 5% of the NAV of the fund and around time of dividend payments these demands can be even greater (see Figure 2). The demand may be too great to close the position by buying or selling the index component stocks, especially for smaller cap funds such as the Russell 2000 iShare. Hence market makers may take large long or short positions in the funds onto their own account, overnight or over a few days until the open position is offset by an opposite demand or supply of the ETFs from investors. In that case they may consider taking out a short-term futures hedge, as discussed in the previous section. However these market makers as well as other traders may think about more imaginative and efficient hedging than simply covering each position with its own future, especially as the net demand is quite heterogeneous (see Table 1) and long-short positions may often be taken on correlated funds.

TABLE 7: DAILY RETURNS AND MISPRICING CORRELATIONS

2001-2002	SPY	QQQQ	DIA	IWM
SPY	1	0.826	0.957	0.885
QQQQ	0.2560	1	0.731	0.801
DIA	-0.1980	-0.1163	1	0.837
IWM	0.1727	0.0680	0.0788	1
2003-2004	SPY	QQQQ	DIA	IWM
SPY	1	0.877	0.968	0.846
QQQQ	0.2588	1	0.820	0.841
DIA	0.2186	-0.0879	1	0.781
IWM	0.2642	-0.0786	0.2115	1

For the four ETFs being studied Table 7 shows the daily returns correlations above the diagonal and the mispricing correlation below the diagonal, again divided into our two sub-samples. Daily returns were very highly correlated in both sub-samples, with the highest correlation between the Diamond and the Spider (as expected since they share many common stocks) and the lowest correlation between the Diamond and the other two funds. All returns correlations are very highly significant, but there is much less correlation in the basis risks of different ETFs: mispricing correlation was relatively small especially during 2001-2002. However during 2003-2004 the only pairs that do not have significant correlation in basis risks are the Diamond and the Cubes, and the Diamond and the Russell iShare.¹⁵ Therefore, a natural question to ask before deciding on the futures hedge is: can the basis risk from a long position on one fund be effectively

¹⁵ Significance is based on a t -ratio of $\frac{r\sqrt{n-2}}{\sqrt{1-r^2}}$ where r is the sample correlation and n is the number of observations (484 in our case). For instance a correlation of 0.2 has a t -ratio of 4.48, which is very highly significant.

offset by a short position on a correlated fund? Our results in this section will show that during the last two years of the sample the Diamond could be hedged almost as effectively with the Spider as with the DJIA future. Similarly, the Diamond is almost as good a hedge for the Spider as the S&P500 future.

Figure 4 compares the exponentially weighted moving average volatility of two hedged portfolios: the Diamond hedged 1:1 with the DJIA future and the minimum variance cross-hedged portfolio of the Diamond with the Spider. We have only depicted the GARCH minimum variance hedge portfolio volatility here, as the OLS and EWMA hedged portfolios are very similar and their volatilities are difficult to distinguish on a graph. From mid 2002 until the end of the period the two ETFs were very highly correlated as a result there are several instances where the minimum variance cross-hedged portfolio has lower volatility than the 1:1 futures hedged portfolio.

[Figure 4]

When hedging with equity index futures it normally makes little difference whether we estimate regression based minimum variance hedge ratios using the spot or the future return as the dependent variable. One hedge ratio is simply the other hedge ratio multiplied by the relative variance and since spot and futures have a relative volatility near to unity the two estimated hedge ratios are very similar. But with the volatility differences between funds noted above (Table 2) the choice of dependent variable is important. It is straightforward to show that one should take the fund having lower returns volatility as the dependent variable to obtain the hedged portfolio with the smaller variance.

Therefore consider two funds with market prices X_1 and X_2 with Fund 1 having the lower returns variance. When a long position on Fund 1 at time t is hedged by selling $\beta_t(\tau)$ units of Fund 2 and the position will be closed at time $t + \tau$, the minimum variance hedge ratio for a hedge of duration τ is:

$$\beta_t^*(\tau) = \frac{\sigma_{12,t}(\tau)}{\sigma_{2,t}^2(\tau)} \quad (8)$$

where $\sigma_{12,t}(\tau)$ and $\sigma_{2,t}^2(\tau)$ denote the returns covariance and the variance of the returns on Fund 2 respectively. As in the previous section we use OLS, EWMA and GARCH hedge ratios following the methods described in the appendix and generating out-of-sample returns series as before.¹⁶ The results are given in Table 8.

¹⁶ Over the entire period there is very weak evidence of cointegration between the diamond and the Spider, with Johansen (1990) trace and maximal eigenvalue tests being significant at 10% and no other fund pairs were found to be cointegrated. Hence we do not include an error correction term in the conditional mean equation for the GARCH hedge ratios and in the Tables we report results using a bivariate vector autoregression as the conditional mean equation (VAR-GARCH)

TABLE 8: CROSS HEDGED PORTFOLIO CHARACTERISTICS**(A) RETURNS VOLATILITY**

2001/2	DIA-SPY	DIA-QQQQ	DIA-IWM	SPY-QQQQ	SPY-IWM	QQQQ-IWM
1:1 ETFs	7.16%	35.07%	14.35%	31.49%	12.17%	31.95%
OLS	7.01%	15.65%	13.35%	13.01%	11.57%	28.52%
EWMA	6.84%	15.47%	13.51%	12.93%	11.78%	29.15%
VAR-GARCH	6.97%	15.53%	13.56%	12.82%	11.63%	28.63%
2003/4	DIA-SPY	DIA-QQQQ	DIA-IWM	SPY-QQQQ	SPY-IWM	QQQQ-IWM
1:1 ETFs	3.54%	12.83%	11.72%	11.24%	10.10%	11.58%
OLS	3.41%	7.78%	8.12%	6.81%	7.15%	11.35%
EWMA	3.46%	7.85%	7.68%	6.88%	6.72%	11.53%
VAR-GARCH	3.41%	7.81%	8.08%	6.83%	7.10%	11.30%

(B) EDERINGTON EFFECTIVENESS, E

2001/2	DIA-SPY	DIA-QQQQ	DIA-IWM	SPY-QQQQ	SPY-IWM	QQQQ-IWM
1:1 ETFs	91.30%	-108.86%	65.01%	-62.98%	75.67%	56.84%
OLS	91.64%	58.40%	69.75%	72.20%	78.01%	65.62%
EWMA	92.06%	59.36%	68.99%	72.54%	77.21%	64.07%
GARCH	91.74%	59.04%	68.76%	73.00%	77.77%	65.35%
2003/4	DIA-SPY	DIA-QQQQ	DIA-IWM	SPY-QQQQ	SPY-IWM	QQQQ-IWM
1:1 ETFs	93.33%	12.42%	26.95%	37.42%	49.42%	70.71%
OLS	93.80%	67.81%	64.88%	77.03%	74.69%	71.85%
EWMA	93.62%	67.16%	68.62%	76.54%	77.66%	70.99%
VAR-GARCH	93.82%	67.51%	65.26%	76.88%	75.02%	72.14%

(C) RETURNS SKEWNESS

2001-2002	DIA-SPY	DIA-QQQQ	DIA-IWM	SPY-QQQQ	SPY-IWM	QQQQ-IWM
1:1 ETFs	-0.3910	-0.2816	0.1791	-0.3359	0.2590	0.5303
OLS	-0.6455	-0.3864	-0.0807	-0.0395	0.2629	0.4589
EWMA	-0.3295	-0.2397	0.1532	0.0388	0.2645	0.4590
VAR-GARCH	-0.6318	-0.3402	0.0988	-0.1140	0.1360	0.4609
2003-2004	DIA-SPY	DIA-QQQQ	DIA-IWM	SPY-QQQQ	SPY-IWM	QQQQ-IWM
1:1 ETFs	0.1689	0.2796	0.2173	0.2438	0.1738	0.1156
OLS	0.1044	0.2312	0.0578	0.2708	-0.1186	0.1167
EWMA	0.0624	0.2223	0.0000	0.2305	-0.1032	0.2011
VAR-GARCH	0.0964	0.2001	0.0483	0.2655	-0.1359	0.1051

(D) RETURNS EXCESS KURTOSIS

2001-2002	DIA-SPY	DIA-QQQQ	DIA-IWM	SPY-QQQQ	SPY-IWM	QQQQ-IWM
1:1 ETFs	2.9626	2.8816	1.0736	2.5304	0.7603	2.9801
OLS	4.6366	2.8401	1.4895	1.5410	0.7409	2.1715
EWMA	2.1480	1.6074	1.4156	1.8307	0.6908	2.2528
VAR-GARCH	4.8158	2.1806	2.6412	1.4157	0.6174	2.3349
2003-2004	DIA-SPY	DIA-QQQQ	DIA-IWM	SPY-QQQQ	SPY-IWM	QQQQ-IWM
1:1 ETFs	0.5414	0.3937	0.1846	0.8378	-0.0037	0.6137
OLS	0.5184	1.4619	0.8069	2.1169	0.8373	0.2102
EWMA	0.4334	1.4188	0.1524	2.1707	0.5532	0.5789
VAR-GARCH	0.5226	1.5327	0.8300	2.0642	0.8312	0.2048

(E) CE OF RETURNS: $\lambda = 10\%$

2001-2002	DIA-SPY	DIA-QQQQ	DIA-IWM	SPY-QQQQ	SPY-IWM	QQQQ-IWM
1:1 ETFs	435.91	-42,037.44	-1,742.05	-26,960.90	-1,866.94	-31,462.94
OLS	121.14	-3,040.23	-1,992.25	-1,926.09	-1,933.14	-19,014.49
EWMA	99.17	-2,740.53	-1,799.24	-1,904.83	-1,829.84	-20,445.63
VAR-GARCH	93.17	-2,748.44	-2,102.97	-1,826.54	-2,166.59	-19,650.63
2003-2004	DIA-SPY	DIA-QQQQ	DIA-IWM	SPY-QQQQ	SPY-IWM	QQQQ-IWM
1:1 ETFs	-377.26	-2,269.93	-2,302.05	-1,678.21	-1,721.26	-1,173.21
OLS	-263.09	-433.86	-1,057.10	-149.96	-806.36	-1,424.13
EWMA	-247.20	-489.29	-738.87	-258.45	-475.35	-918.78
VAR-GARCH	-256.53	-462.65	-1,036.31	-171.36	-793.41	-1,412.61

(F) CE OF P&L: $\lambda = 500$

2001-2002	DIA-SPY	DIA-QQQQ	DIA-IWM	SPY-QQQQ	SPY-IWM	QQQQ-IWM
1:1 ETFs	-939.34	-958,078.78	-6,207.99	-517,469.35	-2,377.03	-230,270.23
OLS	-1022.64	-16,542.61	-5,243.98	-5,182.73	-1,999.95	-100,880.10
EWMA	-576.48	-11,684.06	-5,144.67	-5599.31	-1,986.96	-114,535.83
VAR-GARCH	-1,002.97	-14,530.94	-6,500.93	-4,880.93	-1,865.17	-114,082.51
2003-2004	DIA-SPY	DIA-QQQQ	DIA-IWM	SPY-QQQQ	SPY-IWM	QQQQ-IWM
1:1 ETFs	70.34	-3,007.58	-1,638.18	-2,181.25	-912.01	-3,829.24
OLS	73.11	-591.69	-529.41	-393.88	-314.87	-2,881.21
EWMA	71.38	-590.29	-382.13	-390.43	-234.16	-3,826.85
VAR-GARCH	73.09	-605.30	-521.44	-387.09	-300.96	-2,802.94

From Tables 8(A) and 8(B) we see that hedging the Diamond with the Spider is much more effective for variance reduction than cross hedging any of the other ETFs. Given the highly significant correlation in their mispricing series this is to be expected. The minimum variance Diamond – Spider hedge achieves over 92% effectiveness for variance reduction during 2001-2002 based on the EWMA hedge ratio and nearly 94% effectiveness in 2003-2004 based on the VAR-GARCH hedge ratio. Note that in this case the 1:1 long-short hedge performs about as well as minimum variance hedging, but that is not true for most of the other ETF pairs. For instance a 1:1 hedge of the Diamond with the Cubes is only 12.42% effective during 2003-2004 but using a minimum variance hedge ratio over the same period is over 67% effective. And during 2001-2002 a long position on the Diamond matched with an equal short position on the Cubes actually increased the variance of an un-hedged position on the Diamond, yet minimum variance hedging with the Cubes is almost 60% effective. Clearly minimum variance hedging provides a much greater variance reduction than the 1:1 hedge. However it is not possible to conclude which of the three minimum variance hedge ratios provides the most efficient variance reduction in all cases. The most effective hedges, picked out in bold in Table 8(B) could be any of the minimum variance hedges, depending on the ETF pair and the sample period.

Tables 8(C) and 8(D) report the skewness and excess kurtosis of the cross-hedge portfolio returns and it is here that the potential gains from cross-hedging ETFs appear most promising. Futures hedging produced portfolios with highly significant negative skewness and excess kurtosis in most cases (Tables 5 and 6). However the cross-hedged portfolios have returns that are much closer to normality: during the 2003-4 period in particular the hedged portfolio returns exhibit low levels of skewness and excess kurtosis. The skewness and excess kurtosis are still significant in most cases, but they are much less than for returns based on futures hedging even when the cross-hedge is effectively reducing the variance.

Tables 8(E) and 8(F) report the certainty equivalents of the ETF cross-hedged portfolios in each subsample, again based on both returns and P&L and using the same values of risk tolerance as previously, which are low enough to capture the higher moment effects. Note the high values for the CE of the Diamond-Spider hedge, based on returns in 2001-2 and on P&L in 2003-4 indicate that it can be more attractive to cross-hedge these ETFs than it is to hedge each of them individually using their own futures. The CE criterion also favours a minimum variance hedge ratio more often than not, so the results support the conclusions drawn from examining variance reduction. We conclude that whilst there are substantial gains to be made from using minimum variance hedge ratios for cross-hedging ETFs it is not possible to distinguish which minimum variance hedge ratio is preferred.

VI HEDGING PORTFOLIOS OF ETFs WITH INDEX FUTURES

The natural cross hedging of some ETFs implies that a futures hedged portfolio containing just a few index futures could be almost as efficient (and certainly cheaper) than hedging each ETF in the portfolio with its associated index future. In this section we investigate whether this is indeed the case by constructing various ETF portfolios and comparing the hedge with all four futures with a hedge using only one index future, i.e. the S&P 500 future, this being the most liquid of the four.

We considered the six portfolios shown in Table 9. Portfolio 1 is composed of 1 unit block in each ETF, i.e. 10 shares in the Spider, 100 shares in the Diamond, 40 shares in the Cubes and 5 of the Russell iShare; Portfolios 2, 3 and 4 have long and short positions in different ETFs; Portfolio 5 is long only and compared with Portfolio 1 is tilted toward the Spider; and Portfolio 6 is constructed to have equal amounts invested in each ETF.

TABLE 9: NUMBER OF UNIT BLOCKS IN CANDIDATE PORTFOLIOS

	SPY	DIA	QQQQ	IWM
Portfolio 1	1	1	1	1
Portfolio 2	1	-1	0	0
Portfolio 3	1	-1.5	0	1
Portfolio 4	-1	-1	0.5	0
Portfolio 5	10	1	2	1
Portfolio 6	175	24	75	525

For Portfolio 1, comprising an equal number of unit blocks in all four ETFs, an OLS minimum variance optimization algorithm yields the futures hedge ratios shown in figure 5. The algorithm uses an equally weighted covariance matrix based on the previous one year of daily returns to optimise the futures positions for minimum variance in the hedged portfolio, rolling the sample daily. Note that the futures positions frequently diverge from the equal and opposite positions that would be adopted if these ETFs were separately hedged. This finding, which is related to our previous results on cross-hedging ETFs provokes the question of whether hedging an ETF portfolio using all the associated index futures is the most efficient hedging strategy.

[Figure 5]

We first examine the out-of-sample hedging performance of minimum variance futures hedges and compare this with the 1:1 hedge, both hedges being based on all the relevant index futures. The results are shown in Table 10. Variance reduction effectiveness is extremely high for all portfolios and the Ederington measure always favours the 1:1 futures hedges over OLS. But the hedged portfolio returns have very high excess kurtosis and it is generally higher for the 1:1 hedge portfolio returns than for the OLS hedged portfolio returns. Nevertheless it is only during 2001-2 that the CE criteria favours the OLS

hedge over the 1:1 hedge for most portfolios. In the second period the hedge portfolios all have such low volatility that the high kurtosis in returns has little effect on the CE (recall that in (7) it is the un-normalized skewness and kurtosis that enter the CE approximation).

TABLE 10: PERFORMANCE OF PORTFOLIOS OF ETFs HEDGED WITH ALL FUTURES

2001-2002	Moments of Returns				Performance Measure		
1:1	Average Return	Volatility	Skewness	XS Kurtosis	E	CE Returns	CE Percentile
Portfolio 1	0.61%	1.28%	-1.4241	17.6758	99.76%	52.45	95.90
Portfolio 2	0.76%	2.17%	-1.8840	21.2518	99.21%	47.03	65.51
Portfolio 3	0.77%	2.15%	-1.8229	19.9964	99.23%	48.74	68.57
Portfolio 4	0.77%	2.01%	-1.8233	19.8548	99.27%	53.22	75.64
Portfolio 5	0.56%	1.12%	-0.7931	4.7012	99.82%	49.19	98.66
Portfolio 6	0.46%	1.55%	-0.2463	2.9158	99.66%	33.71	97.03
OLS	Average Return	Volatility	Skewness	XS Kurtosis	E	CE Returns	CE Percentile
Portfolio 1	1.15%	1.58%	-0.4681	10.1134	99.63%	101.86	94.21
Portfolio 2	1.00%	2.39%	-0.1546	12.2118	99.21%	69.03	70.81
Portfolio 3	0.93%	2.37%	-0.1913	12.0597	99.23%	62.12	71.81
Portfolio 4	0.56%	2.30%	-0.1769	13.1837	99.04%	27.02	73.75
Portfolio 5	0.87%	1.49%	-0.7720	7.3143	99.68%	75.73	95.86
Portfolio 6	1.32%	1.89%	-0.4116	2.2403	99.49%	113.10	94.17
2003-2004	Moments of Returns				Performance Measure		
1:1	Average Return	Volatility	Skewness	XS Kurtosis	E	CE Returns	CE Percentile
Portfolio 1	-0.25%	0.77%	-0.6541	6.1234	99.71%	-28.17	99.54
Portfolio 2	-0.20%	1.18%	-0.2836	7.1403	99.26%	-26.81	98.36
Portfolio 3	-0.19%	1.17%	-0.3117	7.1441	99.27%	-26.32	98.37
Portfolio 4	-0.20%	1.09%	-0.3058	7.1266	99.35%	-26.19	98.71
Portfolio 5	-0.28%	0.73%	-1.7788	9.5074	99.75%	-30.39	99.49
Portfolio 6	-0.32%	1.00%	-0.4440	5.3932	99.59%	-37.59	99.08
OLS	Average Return	Volatility	Skewness	XS Kurtosis	E	CE Returns	CE Percentile
Portfolio 1	-0.64%	1.03%	-0.2995	3.8506	99.48%	-68.95	99.11
Portfolio 2	-0.60%	1.55%	0.0140	4.0879	99.26%	-71.97	96.91
Portfolio 3	-0.58%	1.56%	-0.0260	3.7307	99.27%	-70.40	96.95
Portfolio 4	-0.36%	1.47%	0.0050	3.7147	98.80%	-47.01	97.51
Portfolio 5	-0.49%	0.96%	-0.6801	4.5933	99.56%	-53.31	99.18
Portfolio 6	-0.57%	1.36%	-0.0943	2.9556	99.24%	-66.45	98.19

Finally we consider hedging each portfolio with only the S&P500 future. The OLS hedge portfolio characteristics are shown in Table 11. The efficiency for variance reduction is lower than when hedging with all four futures, especially for the long-short portfolios (portfolios 2, 3 and 4). Indeed the single futures hedge is clearly better for long only portfolios (portfolios 1, 5 and 6). During the first period the

hedge does not effectively neutralize the mean returns and these dominate the CE criteria. During the second period the means returns are much lower (except for portfolio 6) and the kurtosis is exceptionally low. The most encouraging results are for the long only portfolios 1 and 5 during 2003-4: the single S&P 500 future hedge is highly efficient for variance reduction, both skewness and kurtosis are much lower than when the portfolio is hedged using all four futures, and the certainty equivalent of P&L is almost as high as when these portfolios are hedged with all four futures.

TABLE 11: PERFORMANCE OF PORTFOLIOS OF ETFs HEDGED WITH S&P500 FUTURE

	Moments of Returns				Performance Measure		
2001-2002	Average Return	Volatility	Skewness	XS Kurtosis	E	CE Returns	CE P&L
Portfolio 1	3.99%	4.31%	-0.3899	2.7097	97.22%	292.52	- 27.74
Portfolio 2	-11.34%	9.02%	-0.3913	4.9509	86.32%	-1808.24	-1835.90
Portfolio 3	-11.42%	8.98%	-0.3488	4.8885	86.53%	-1800.72	-1755.78
Portfolio 4	-19.03%	10.11%	-0.4590	4.1644	81.50%	-2803.70	-2194.28
Portfolio 5	6.99%	3.24%	-0.3986	2.5213	98.47%	642.03	57.70
Portfolio 6	10.61%	4.70%	-0.1391	1.6382	96.86%	938.51	-61.15
2003-2004	Average Return	Volatility	Skewness	XS Kurtosis	E	CE Returns	CE P&L
Portfolio 1	-1.49%	2.39%	0.0957	0.3418	97.20%	-177.81	92.61
Portfolio 2	-0.91%	5.15%	0.2719	0.5615	85.87%	-228.25	-24.99
Portfolio 3	0.12%	5.27%	0.2983	0.6374	85.17%	-131.11	-40.66
Portfolio 4	-1.00%	5.70%	0.3246	0.5787	82.09%	-268.45	-80.32
Portfolio 5	-1.07%	1.56%	-0.0747	0.5289	98.84%	-119.71	97.93
Portfolio 6	-6.46%	3.47%	-0.4262	0.7660	95.06%	-710.85	70.34

VII CONCLUSIONS

The basis risk of equity indices is now very small indeed and a natural question to address is whether minimum variance hedging of equity indices remains an interesting research topic. A considerable number of recent papers investigate the effectiveness of hedging equity indices using minimum variance hedge ratios, yet many of these studies are based on daily data where non-synchronous closing prices could bias results. This is not a concern here as we use ETFs, which close at the same time as the future.

The first empirical results in this paper compared the out-of-sample performance of OLS regression, exponentially weighted moving averages and ECM-GARCH hedging models with the naïve futures hedge in which one equivalent unit of the ETF is hedged with one short position in its index future. The variance reduction of 1:1 hedging was found to be at least as great as, and often greater than that achieved by minimum variance hedging. Whilst hedging an ETF is more efficient than hedging the spot index, because of lower trading costs and less dividend uncertainty, it is likely that a similar conclusion could be

drawn from daily hedging of the spot index. That is, basis risk is now so low that it is unlikely that minimum variance hedge ratios would have variance efficiency greater than that of the 1:1 futures hedge.

Further results analysed the cross-hedging of ETFs, i.e. the extent to which netting opposite positions on ETFs could reduce variance prior to futures hedging. This issue is of concern to tax arbitrage investors and to market makers in ETFs that we find can have large but uncorrelated creation or redemption demands on different ETFs at the end of each day, especially around the time of dividend payments. If ETF shares are not redeemed or created, we found that the prior netting of ETFs according to the minimum variance hedge ratio can considerably reduce the costs of overnight futures hedging. In this case there is no doubt that minimum variance hedging is more efficient than simply netting equal and opposite positions in two ETFs. A surprising degree of variance reduction is possible and, moreover, this type of netting produces portfolios with much lower skewness and kurtosis than futures hedged portfolios. It was, however, not possible to identify any single model that provides the best cross-hedge in each case: this depends on the data period and the performance criterion used.

The encouraging results on cross-hedging led us to an empirical investigation of hedging portfolios constructed from the four ETFs, using first all four index futures and then only the S&P 500 future (this being the most liquid of the four). Although the portfolios hedged with all relevant futures are highly efficient for variance reduction the OLS hedged portfolio based only on the S&P 500 futures contract has a much lower kurtosis. As a result the utility achieved by the single futures hedge can almost as great as the utility based on hedging with all futures. This, combined with the obvious reduction in transaction costs, could make single futures hedging of ETF portfolios an attractive proposition for ETF market makers and short-term investors.

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APPENDIX: TIME-VARYING MINIMUM VARIANCE HEDGE RATIOS

Consider a cash position in the ETF at time t that is hedged by selling $\beta_t(\tau)$ units of a T -maturity future with market price F_t assuming the position will be closed at time $t + \tau$, with $0 < \tau < T$. We have adjusted the ETF prices S_t for the cash account and dividends and hence base the optimal hedge ratio on the τ -period index return and the futures 'return', defined as:

$$R_t^S(\tau) = \frac{(S_{t+\tau} - S_t)}{S_t} \text{ and } R_t^F(\tau) = \frac{(F_{t+\tau} - F_t)}{S_t}$$

Denote the variance of $R_t^F(\tau)$ at time t by $\sigma_{F,t}^2(\tau)$ and the covariance between $R_t^S(\tau)$ and $R_t^F(\tau)$ by $\sigma_{SF,t}(\tau)$. Then the minimum variance hedge ratio for a hedge of duration τ is given by:

$$\beta_t^*(\tau) = \frac{\sigma_{SF,t}(\tau)}{\sigma_{F,t}^2(\tau)} \quad (\text{A.1})$$

With this hedge ratio the return on the hedged portfolio between time t and time $t + \tau$ is

$$R_t^S(\tau) - \beta_t^*(\tau) R_t^F(\tau)$$

and the variance of this return is

$$\sigma_t^*(\tau) = \sigma_{S,t}^2(\tau) (1 - \rho_{SF,t}^2(\tau)),$$

where $\sigma_{S,t}^2(\tau)$ and $\rho_{SF,t}(\tau)$ denote the variance of the τ -period index return and the correlation between the τ -period returns on the index and the future at time t .

The simplest of all the hedge ratios considered in this study – apart from the so-called 'naïve' 1:1 ratio – is the minimum variance hedge ratio (A.1) estimated using OLS. We also consider time-varying estimates of (A.1) based on an exponentially weighted moving average (EWMA) of the numerator and denominator. By employing these models one faces the ambiguity of estimating a parameter value that is not necessarily 1, although we know that the parameter must be 1 at some point in time (i.e. when the future expires). For this reason we also consider a time-varying parameter model that can also account for the fact the spot and futures are cointegrated and hence adjust the parameter towards 1 as the future approaches expiry.

To model the effect of spot-futures cointegration we include the carry cost in a bivariate error correction model (ECM) for deriving the optimal futures hedge ratio.¹⁷ To see why, take logarithms of (2) giving:

$$\ln F_t^* - \ln S_t = (r - q)(T - t)$$

Hence if the carry cost, $C_t = (r - q)(T - t)$ is stationary the logarithm of the spot price and the logarithm of the fair value of the futures price should be cointegrated with cointegrating vector (1, -1). However the carry cost need not be the most stationary linear combination of the log of the market price

¹⁷ See Ghosh (1993).

of the future and the log of the spot price. Nevertheless since the mispricing of the future relative to its fair value is so small it is reasonable to assume the error correction term in the error correction model is equal to the carry cost. We shall adopt this formulation because it is more intuitive and hence specify the following error correction model:

$$\mathbf{y}_t = \boldsymbol{\mu} + \sum_{i=1}^n \boldsymbol{\Gamma}_i \mathbf{y}_{t-i} + \pi C_{t-n} + \boldsymbol{\varepsilon}_t$$

where \mathbf{y}_t is the vector of τ -period log returns on the future and spot, $\boldsymbol{\varepsilon}_t$ is the vector of unexpected returns to future and spot and $\boldsymbol{\mu}, \pi$ and $\boldsymbol{\Gamma}$ are constants, with

$$\mathbf{y}_t = \begin{pmatrix} r_{F,t}(\tau) \\ r_{S,t}(\tau) \end{pmatrix}, \boldsymbol{\mu} = \begin{pmatrix} \mu_F \\ \mu_S \end{pmatrix}, \pi = \begin{pmatrix} \pi_F \\ \pi_S \end{pmatrix}, \boldsymbol{\Gamma}_i = \begin{pmatrix} \Gamma_{i,F,F} & \Gamma_{i,F,S} \\ \Gamma_{i,S,F} & \Gamma_{i,S,S} \end{pmatrix} \text{ and } \boldsymbol{\varepsilon}_t = \begin{pmatrix} \varepsilon_{F,t} \\ \varepsilon_{S,t} \end{pmatrix}$$

The basis risk that needs to be hedged is then the conditional variation in $\varepsilon_{S,t}$. The time-varying optimal hedge ratio between spot and futures prices is given by:

$$\tilde{\beta}_t^*(\tau) = \frac{\tilde{\sigma}_{SF,t}(\tau)}{\tilde{\sigma}_{F,t}^2(\tau)} \quad (\text{A.2})$$

where $\tilde{\sigma}_{SF,t}(\tau)$ and $\tilde{\sigma}_{F,t}^2(\tau)$ denote the conditional covariance of the unexpected returns to spot and future, and the conditional variance of the unexpected future return respectively. We model time-variation in a fully conditional bivariate GARCH framework, assuming that

$$\boldsymbol{\varepsilon}_t | \boldsymbol{\Omega}_{t-1} \square N(0, \mathbf{H}_t)$$

where $\boldsymbol{\Omega}_{t-1}$ denotes the information set at time $t-1$ and

$$\mathbf{H}_t = \begin{pmatrix} \tilde{\sigma}_{F,t}^2(\tau) & \tilde{\sigma}_{SF,t}(\tau) \\ \tilde{\sigma}_{SF,t}(\tau) & \tilde{\sigma}_{S,t}^2(\tau) \end{pmatrix}$$

The most sophisticated minimum variance hedge ratio estimates combines ECM with GARCH models instead of EWMA. We use a variety of bivariate GARCH(1,1) parameterisations of the dynamics of \mathbf{H}_t , each of which has been well documented, including BEKK specifications (Engle and Kroner, 1995) and the dynamic conditional correlation model of Engle (2002). The BEKK specification ensures positive definiteness while imposing cross equation restrictions (e.g. the scalar BEKK imposes that persistence in volatility and correlation are the same). The t -BEKK replaces the conditional normality assumption with that of conditionally t -distributed error terms. The dynamic conditional correlation (DCC) model is an extension of the constant conditional correlation estimator of Bollerslev (1990) where the correlation matrix has time-varying estimates based on a constrained form of the ‘diagonal vech’ GARCH parameterization.

Figure 1: Total Market Value of ETF Trusts (in billion USD)

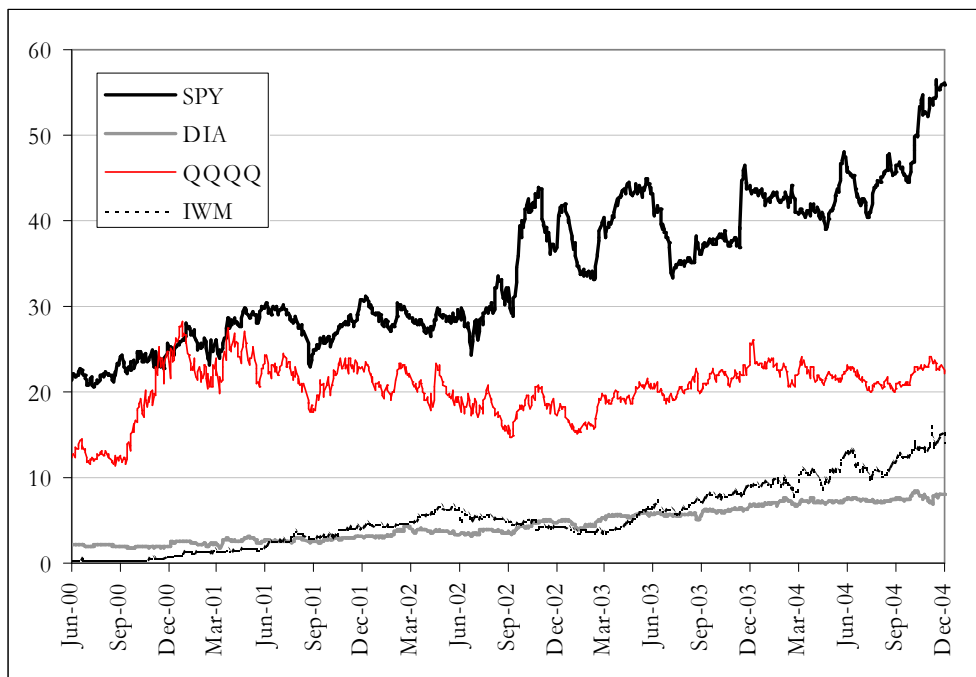


Figure 2: Net Daily Creations and Redemptions as a Percentage of the NAV of the Fund

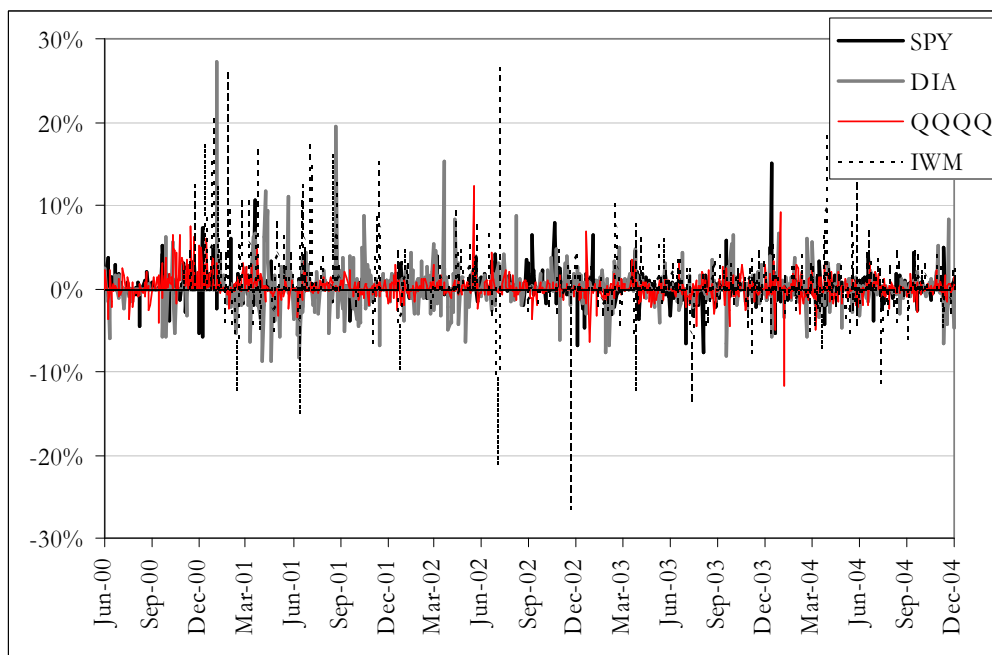


Figure 3: ‘Mispricing’ of Future Relative to ETF Market Price.

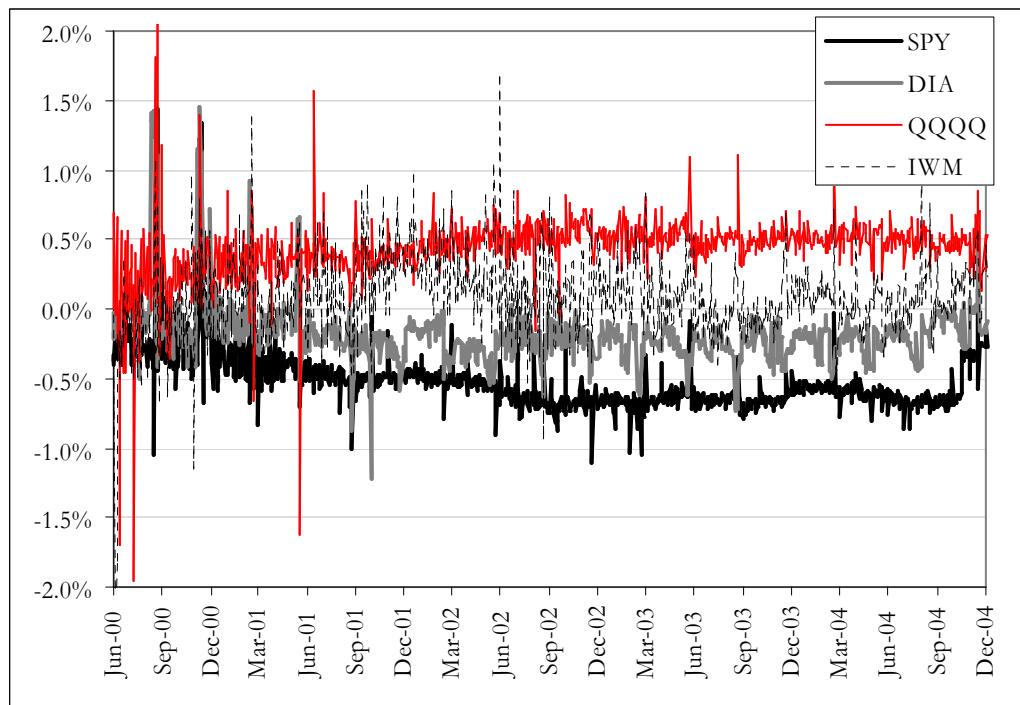


Figure 4: Naïve Futures and Minimum Variance Cross-Hedged Portfolio Volatility
Diamond – DJIA Future & Diamond – Spider

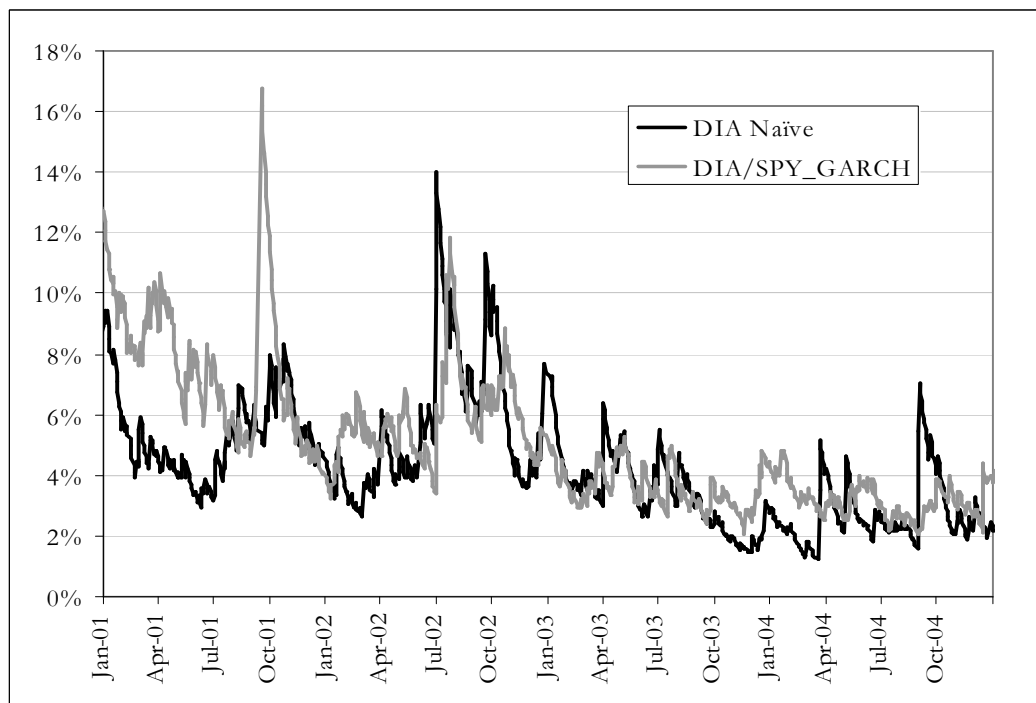


Figure 5: Minimum Variance Hedge Ratios for Portfolio 1 (Equal Shares in the ETFs)

